Thermodynamics and Statistical Physics

Part I – Thermodynamics

Exam T1

Friday, September 24 2021, 9:00-11:00, Aletta Jacobshal 01

The total number of points that can be reached in this exam is 90.

Final grade = (points/10) + 1.

1) Assume that a home requires 1500 m³ of natural gas per year for heating and cooking. If the gas consists of methane (CH_4) only and it behaves like an ideal gas, what is the mass of the gas in kg at T=20°C and p=1 bar and what is the amount in mole under these conditions? By how many percent would the consumed volume change, if the owners would leave the room with the gas-meter at T=15°C?
   M_H=1 g/mol, M_C=12 g/mol. **(20 pt)**
2) Assume a model system of 50 electrons with random spins either ↑ or ↓. What is the difference in probability between the outcomes 25↑:25↓, 24↑:26↓ and 1↑:49 ↓? Use this model system to explain the concept of macrostates and microstates. **(20 pt)**
3) Show that the speed distribution of ideal gas particles that escape through a small circular opening (effusion) is not the Maxwell Boltzmann distribution and that effusion selects faster particles. You may use the fact that the fraction of particles moving under an angle θ (i.e. in the interval $(\theta, \theta + d\theta)$) with respect to the normal of the opening is given by the expression.

$$\frac{1}{2}\sin(\theta)d\theta$$

Furthermore, the number of particles moving with speed v (interval $(v, v + dv)$) is given by:

$$nf(v)dv$$

with n being the particle density and $f(v)$ being the Maxwell Boltzmann distribution.

Use a Sketch! **(20 pt)**

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4) Consider an ensemble consisting entirely of atoms that can be described as 2-level systems (with ground state E_0=0 eV and a single excited state E_1=3 eV). All particles are in thermal equilibrium at temperature T. Use the concept of the canonical ensemble to determine the ratios of particles being in the ground state and in the excited state for the following temperatures.

- $T=10^0$ K
- $T=10^2$ K
- $T=10^4$ K
- $T=10^6$ K

Excitation energies of a few eV are typical for electronic excitation in atoms and molecules. What do the determined ratios imply for a gas at room temperature, the plasma on the surface of the sun and the plasma in the core of the sun (estimate typical temperature ranges for these environments)? **(20 pt)**

5) Give the thermodynamic definition of the temperature in terms of the number of microstates and the energy of the system. **(10 pt)**

Constants:

Avogardro's number:	$N_A=6.02\ 10^{23}\ \mathrm{mol}^{-1}$
Boltzmann constant:	$k_B=1.381\ 10^{-23}$ J\K
Gas constant:	$R=8.31\ \mathrm{JK}^{-1}\mathrm{mol}^{-1}$
Atomic mass unit (u):	$m_u=1.67\ 10^{-27}$ kg
Electronvolt:	1 eV=$1.6\ 10^{-19}$ J

Integrals:

n	$\int_0^\infty dx\, x^n e^{-ax^2}$ (a > 0)
0	$\frac{1}{2}\sqrt{\left(\frac{\pi}{a}\right)}$
1/2	$\frac{0.6127}{a^{3/4}}$
1	$\frac{1}{2a}$
3/2	$\frac{0.4532}{a^{5/4}}$
2	$\frac{1}{4a}\sqrt{\frac{\pi}{a}}$

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